Fall 2013 **Math 601** Name: ______________________

Quiz 2

Question 1. (15 pts)

(a) (10 pts) Suppose

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 4 \\ 0 & -1 & 2 \end{bmatrix}$$

Use Gaussian elimination to find A^{-1}.

Solution:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 2 & 0 & 4 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{(-2)R_1+R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -2 & -2 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1/2 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & -1 & 1/2 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & -1 & 1/2 & 0 \\ 0 & 0 & 1 & -1 & 1/2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3+R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1/2 & 0 \\ 0 & 1 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1/2 & 1 \end{array}\right]$$

$$\xrightarrow{(-2)R_3+R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -1/2 & -2 \\ 0 & 1 & 0 & -2 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1/2 & 1 \end{array}\right]$$

So

$$\boxed{A^{-1} = \begin{bmatrix} 2 & -1/2 & -2 \\ -2 & 1 & 1 \\ -1 & 1/2 & 1 \end{bmatrix}}$$

(b) (5 pts) use Part (a) to solve the linear system

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

Solution:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

Question 2. (5 pts)

Recall that a square matrix A is invertible if there exists a matrix B such that

$$AB = I_n = BA$$

where I_n is the identity matrix of size $n \times n$. In this case, we call B an inverse of A. Now show that if A is invertible, then A has a unique inverse.

Solution: Equivalently, the question can be stated in the following way. Suppose B and C satisfy

$$AB = I_n = BA,$$

$$AC = I_n = CA,$$

then we need to show that $B = C$.

To prove this, we notice that

$$BAC = (BA)C = I_nC = C$$

and

$$BAC = B(AC) = BI_n = B.$$

It follows that $B = BAC = C$.